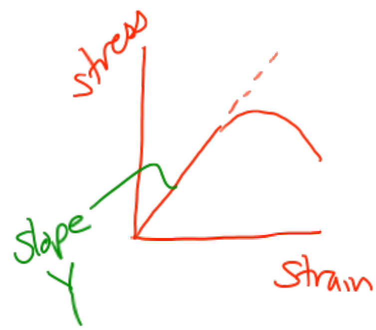
October 6

Young's Modulus



$$Y = \frac{F_\perp / A}{\Delta L / L} \quad \text{macroscopic}$$

$$= \frac{k_{s,i} \cancel{s} / d^{\cancel{2}}}{\cancel{s} / \cancel{d}}$$

$$= k_{s,i} / d \quad \text{microscopic}$$

$$\text{stress} = Y \, \text{strain}$$

$$Y = \frac{\text{stress}}{\text{strain}}$$

Odd tables interatomic spring const for gold

Y on line

Even ________ " ________ lead

Odd $Y = 7.8\times10^{10}\ N/m^2$

$$k_{s,i} = Yd = \left(7.8\cancel{\times10^{10}}\ \frac{N}{m^2}\right)\left(2.6\cancel{\times10^{-10}}\ m\right) = 20.\ \frac{N}{m}$$

Even $Y = 1.6\times10^{10}\ N/m^2$

$$k_{s,i} = Yd = \left(1.6\times10^{10}\ \frac{N}{m^2}\right)\left(3.1\times10^{-10}\ m\right) = 5\ \frac{N}{m}$$

You hang 1000 kg on 30 m steel wire

that is 3 cm in diameter

$$Y = 2\times10^{11}\ N/m^2$$

How much does it stretch?

$$Y = \frac{F/A}{\Delta L/L}$$

$$\Delta L = \frac{FL}{AY} = \frac{(1000\,kg)(9.8\,m/s^2)(30\,m)}{\pi(1.5\times10^{-2}\,m)^2(2\times10^{11}\,N/m^2)} = 2.1\times10^{-3}\,m$$

$$= 2.1\,mm$$

More about springs

guess that

angular frequency

$$x = A\cos(\omega t)$$

amplitude

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{net}}$$

$$\frac{dp_x}{dt} = F_{\text{net}\,x} = -k_s x$$

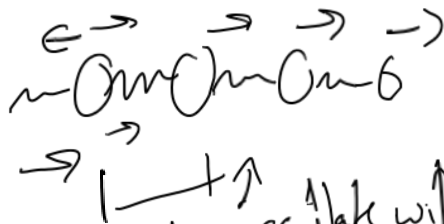
$$m\frac{dv_x}{dt} = m\frac{d^2x}{dt^2} = m\frac{d^2\left(A\cos(\omega t)\right)}{dt^2} = m\frac{d\left(-A\omega\sin(\omega t)\right)}{dt} = m\left(-A\omega\cos(\omega t)\right) = -k_s x$$

$$m\left(-A\omega^2\cos(\omega t)\right) = -k_s \underbrace{A\cos(\omega t)}_{x} \quad \text{so} \quad m\omega^2 = k_s \quad \text{and} \quad \omega = \sqrt{\frac{k_s}{m}} = \frac{2\pi}{T}$$

period

Also useful to define $f \equiv \frac{1}{T}$ so $\omega = 2\pi f$

Speed of sound in solids



d   oscillate with frequency

$$v \sim d\omega \qquad \omega = \sqrt{\frac{k_{Si}}{m}}$$

$$\sim \sqrt{\frac{k_{Si}}{m_{atom}}}\, d_{atom}$$

Odd speed of sound gold

even — " — lead

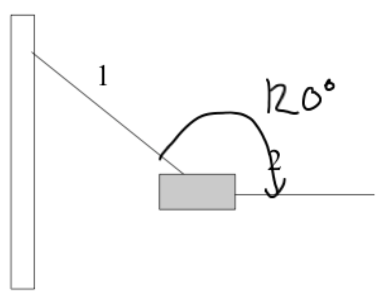
$$v = d\sqrt{\frac{k_{Si}}{m_{atom}}}$$

$$v = d\sqrt{\frac{k_{s,i}}{m}}$$

$$= 2.6\times10^{-10}\,m\sqrt{\frac{20.3\,N/m}{3.3\times10^{-25}\,kg}} = 2040\,m/s \quad Au$$

$$= 3.1\times10^{-10}\,m\sqrt{\frac{5.0\,N/m}{3.5\times10^{-25}\,kg}} = 1170\,m/s \quad Pb$$

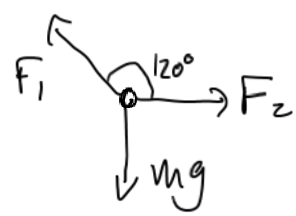
Statics Calculation



$m = 36 g$

System block

$\vec{F}_1 = ?$ $\vec{F}_2 = ?$



$$\frac{d\vec{p}}{dt} = \vec{F}_{net} = 0$$

$$\langle 0,0,0 \rangle = \vec{F}_{net} = \vec{F}_{grav} + \vec{F}_1 + \vec{F}_2$$

$$\langle 0,0,0\rangle = \vec{F}_1 + \vec{F}_2 + F_{grav}$$

$$\vec{F}_{grav} = \langle 0, -mg, 0\rangle$$

$$\vec{F}_1 = |\vec{F}_1|\,\hat{F}_1 = |\vec{F}_1|\langle \cos(120^\circ), \cos(30^\circ), \cos(90^\circ)\rangle$$

$$\vec{F}_2 = |\vec{F}_2|\,\hat{F}_2 = |\vec{F}_2|\langle 1,0,0\rangle$$

x: $|\vec{F}_1|\cos(120^\circ) + |\vec{F}_2| = 0$

y: $|\vec{F}_1|\cos(30^\circ) - mg = 0$

$$|\vec{F}_1| = \frac{mg}{\cos(30^\circ)}$$

$$|\vec{F}_2| = -\frac{\cos(120^\circ)}{\cos(30^\circ)}mg$$

Now the monster: A big road sign is supported by a pole and cable as shown. How much does the supporting cable stretch? If a stupid bird flies into the top center of the sign, how long will it take for that sound to travel along the cable and reach the pole? Don't actually do the problem, but do the G and O steps in the GOAL protocol.

A of cable

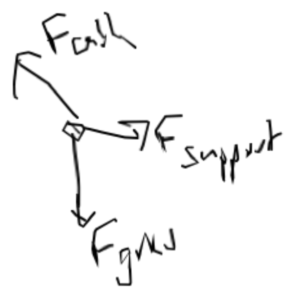
L of cable

m of sign

material of cable

↳ Y, ρ

↳ d, $k_{s,i}$, V



Momentum Principle for Curving Motion

$$\vec{p} = |\vec{p}|\hat{p} \qquad \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(|\vec{p}|\hat{p}\right) = \hat{p}\frac{d|\vec{p}|}{dt} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\left|\frac{d\vec{p}}{dt}\right|\hat{p} = \vec{F}_{\parallel} \quad \text{and} \quad |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \vec{F}_{\perp} \quad \text{or} \quad p\frac{v}{R} = F_{\perp}$$

$\longrightarrow \hat{p}\frac{d|\vec{p}|}{dt}$ in direction of $\vec{p}$ ($\parallel$ to $\vec{p}$)

$|\vec{p}|\frac{d\hat{p}}{dt}$

$\frac{d\hat{p}}{dt}$ is $\perp$ to $\hat{p}$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net} = \vec{F}_{net,\parallel} + \vec{F}_{net,\perp} = \hat{p}\frac{d|\vec{p}|}{dt} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$\frac{pv}{R}$ is force required to move in circle

Geosynchronous Satellite

$$p\frac{v}{R}=F_{\perp}$$

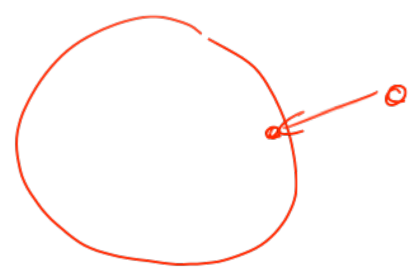
$$\frac{\cancel{m}_{\text{Satellite}}v^2}{\cancel{R}}=G\frac{m_{\text{Earth}}\cancel{m}_{\text{Satellite}}}{R^{\cancel{2}}}\quad \text{so}\quad v^2=G\frac{m_{\text{Earth}}}{R}\quad \text{we know}\quad v=\frac{2\pi R}{T}$$

$$\left(\frac{2\pi R}{T}\right)^2=G\frac{m_{\text{Earth}}}{R}\quad \text{or}\quad \frac{4\pi^2R^2}{T^2}=G\frac{m_{\text{Earth}}}{R}$$

$$R^3=G\frac{m_{\text{Earth}}}{4\pi^2}T^2\quad \text{or}\quad R=\sqrt[3]{G\frac{m_{\text{Earth}}}{4\pi^2}T^2}$$

$$R=\sqrt[3]{\left(6.7\times10^{-11}\,\tfrac{N\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(6\times10^{24}\,\text{kg}\right)}{4\pi^2}\left(86400\text{ s}\right)^2}=\sqrt[3]{7.61\times10^{22}}=4.24\times10^7\,\text{m}\quad \text{or}\quad 26{,}400\text{ mi}$$

$$4.24\times10^7\,\text{m}\ -\ 6.4\times10^6\,\text{m}=3.6\times10^7\,\text{m}\quad \text{or}\quad 22{,}000\ \text{mile altitude}$$



Group A: How fast is Earth traveling such that it maintains a circular orbit around the Sun?

$$\frac{pv}{R} = F_{\perp} = \frac{\cancel{M_\oplus} v^2}{\cancel{R}} = G\frac{\cancel{M_\oplus} M_\odot}{R^{\cancel{2}}} \qquad v = \sqrt{\frac{G M_\odot}{R}} = 3.0\times10^{4}\,\text{m/s}$$

Group B: How fast is the Moon traveling such that it maintains a circular orbit around the Earth?

$$v = \sqrt{\frac{G M_\oplus}{R}} = 1000\,\text{m/s}$$

Group C: How fast is the international space station traveling such that it maintains a circular orbit around the Earth?

R = 360 km + radius of Earth = 360e3 + 6.4e6 = 6.76e6 m

$$v = \sqrt{\frac{G M_\oplus}{R}} = 7700\,\text{m/s}$$